THE IMPACT OF REGULATORY CHANGES ON BANKS' RISKS AND RETURNS IN TAIWAN

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ABSTRACT

The purpose of this paper is mainly to review the existing substitute relationship between capital regulation and deposit insurance system in Taiwan. We conclude a converse relationship exists between variable deposit insurance system and risk-based capital regulation, decrease of one can substitute for increase of another. After examining the impact of regulatory changes on banks' returns and risks during the period 1985 to 2000 in Taiwan, we find out that reducing fixed insurance rate and adopting risk-based capital regulation, the improvement on banks' return to risk ratio is significant. However, it is not significant following the implementation of variable-rate system.

INTRODUCTION

Banks play as roles of adjusting the capital demand and supply in the society. The intermediate manners are mostly meeting the needs in loans with fund deposit from outside. Therefore, the capital ratio is lower in contrast to other

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industries. By nature, banks exist with such high financial leverage risk, the derived interest risk due to the period incompatible between the deposit and loan and the credit risk in the loan process, together they give substantial operational risk.

Banks absorb huge funds from the public to meet the need of enterprise investment and responsible for business guarantee and payment. Once a bank is insolvent, it will certainly cause significant impact on the economic order. So countries all over the world have been placing all kinds of regulations to minimize the probability of bank insolvency. The major regulations include entry regulation, price regulation, business regulation and capital regulation.

The past regulating instruments are unable to match the speed of the progress in information technology and financial information. As a result, they would obstruct the complete competition mechanism and demand the regulators to look for new regulating instruments. The most important among them is riskbased capital regulation, which sets the required capital level based on the asset risk structures of banks. The riskier the assets are, the higher the capital level is required, so as to increase the capacity of the banks in sustaining loss and reduce the probability of bankruptcy. On the other hand, it will protect the depositors by receiving higher compensation against the liquidation of banks.

The other important regulating instrument is deposit insurance. Banks may decide whether to participate in the deposit insurance system. The participating banks are liable to pay a premium on a timely basis, and are subject to financial inspection. When a bank is in bad shape and is bankrupted, the deposit insurance company will compensate the certain deposit amount. Hence the banks reduce the operational risk to a certain level and transfer some part of it to a deposit insurance system. Of course, the banks have to resume the consequence of increase in capital cost. Compared to risk-based capital regulation, both of them will increase the banks' capital cost. Used as means of risk control, if they are considered together then more reasonable regulating policies can be set.

From recent rederegulation of bank capital and deposit insurance of banks in Taiwan, we can find the track on both of them evolving. The deposit insurance system was implemented in September 1985 in Taiwan, with the fixed premium rate set at 0.05%. At that time, the regulator had many restrictions on the establishment and operation of banks, and the banks operated rather conservatively and hardly ever went bankrupt, so not many banks participated in the deposit insurance. Latterly, in order to attract participants, the regulator adopted low rate strategy, and lowered the insurance premium rate to 0.04% and 0.015% in January 1987 and January 1988, respectively.

Following the efforts of Taiwan joining WTO, the paces of financial liberation and internationalization have been accelerating, and the regulator eased the restriction on establishment and operations of banks. The competition between banks are getting more intense, so the operational strategies become more open than ever, and the risks faced by banks are much higher. Risk-based capital regulation policy was executed completely in January 1993, requiring the capital level set in accordance with the risk structure of banks' assets, even variable deposit insurance rate was implemented in January 2000 to control the banks' risks through price mechanism and maintain the financial stability.

Kim and Santomero (1988) and Mei-Ying Liu (1994) apply mean-variance model to explain that uniform capital regulation will induce the bank to possess high-risk assets to offset more cost from higher capital, it causes moral risk. Risk-based capital regulation can control bankruptcy effectively. If the regulator utilizes the ceiling of banks' expected return, as a means of controlling bankruptcy, then setting asset risk weights based on asset risk will make banks with higher asset risk prepare to reimburse higher capital. So it can reduce the inducement of higher operational risk effectively.

Applying option-pricing theory on deposit insurance, Merton (1977) shows that the value of deposit insurance is equal to the value of a European put option. It utilizes option-pricing model to price deposit insurance value. Merton (1977), Keely and Furlong (1990) considered the bank would enhance asset risk and leverage ratio to increase deposit insurance value and maximize equity value. Duan, Moreau and Sealey (1992) found that if fixed insurance rate were adopted then the bank would transfer risk to insurance company. Ronn and Verma (1986) suggested variable insurance rate based on risk adjustment would reduce the inducement of the bank transferring risk to insurance company; it should be a more reasonable policy.

The previous articles mostly propose some regulatory policy, such as riskbased capital regulation and variable insurance rate based on risk adjustment, to lessen the moral hazard under uniform capital regulation and fixed-rate deposit insurance system and decrease the improper social resource subsidy on risky banks. If we consider only the effect of preventing fraud, the relationship between the policies and how to coordinate each other to complete regulatory goal can be neglected. It is convenient using mean-variance model to understand the influence of regulatory policies on bank portfolio decision by graphs. This article will be based on mean-variance model to discuss the effectiveness of deposit insurance and capital regulation on risk reduction and strengthen the placement of assets.

The paper is separated into five sections. The mean-variance model of the bank portfolio decision of Kim and Santomero (1988) is introduced firstly. The

second section analyzes how to derive theoretical risk weight when risk-based capital regulation is adopted as means for controlling banks risks, so as to achieve the goal of regulation. Section 3 discusses how to use deposit insurance system as instrument for risk control. Section 4 discusses how the two regulating instruments are combined together to have advanced control of the probability of bank's insolvency. Section 5 is the result of empirical examination. Finally, Section 6 concludes.

This article examines the effects of bank capital regulation and deposit insurance policies on the banks' return to risk ratios during the period between 1985 and 1999. It took the publicly held banks during the period between 1985 and 1999 with full records of participating in deposit insurance system as samples. In Hypothesis I, we test whether higher fixed insurance premium will worsen the tradeoff between banks' returns and risks. In Hypothesis II and III, we test separately whether implementing risk-based capital regulation or variable insurance premium system will improve the tradeoff between banks' returns and risks. Hypothesis I and II are both supported. However Hypothesis III is not supported.

To sum up, there is a converse relationship between variable deposit insurance system and risk-based capital regulation, decrease of one rate can be substitute for increase of another in theory, but each of them has its own function to the regulator. Both of them can be helping and covering each other to achieve the common regulation target goal, it shall be the most feasible manner.

1. MODEL

Koehn and Santomero (1980) and Kim and Santomero (1988) took banks as the constructors of a portfolio. They discussed the relationship among the components of portfolio, asset, liability, and owners' equity, to decide the optimal capital structure and assets structure. The model minimizes risks under fixed wealth. Capital regulation is given with another constraint equation. A given equity-to-asset ratio k implies a fixed deposit-to-equity ratio (1 - 1/k). It means that:

$$\frac{\min_{X_{1}} \frac{1}{2} \sigma^{2} = \frac{1}{2} \underline{X}' \underline{V} \underline{X}}{\left[1 - \frac{1}{k}, \underline{X}'_{1}\right] \left[\frac{\sigma_{0}^{2}}{\underline{V}_{1}} \frac{\underline{V}'_{1}}{\underline{V}_{2}}\right] \left[\frac{1 - \frac{1}{k}}{\underline{X}_{1}}\right]$$
(1)

subject to

$$E = \left(1 - \frac{1}{k}\right)u_0 + \underline{X}'_1 \underline{U}_1,$$
$$\frac{1}{k} = \underline{X}'_1 e,$$

$$X_1' > 0$$
, and $0 < k \le 1$,

where:

- (1) u_0 and σ_0^2 are the mean and variance of costs of deposits.
- (2) \underline{U}_1 is an $n \times 1$ vector of asset returns $[u_i]$ for i = 1, 2, ..., n.
- (3) V₁ is an n×1 vector of covariance [σ_{0i}] between deposit cost and asset returns. V₂ is an n×n variance-covariance matrix of asset returns [σ_{ij}] for i=1, 2, ..., n and is positive-definite.
- (4) \underline{X}_1 is an $n \times 1$ vector of x_i that is the ratio of the *i*th asset holding, as a proportion of the equity capital, and $\underline{X}_1 > 0$ due to short sale restrictions.
- (5) <u>e</u> is an $n \times 1$ vector with first n 1 elements of 1 and the *n*th element of 0. Therefore, $\underline{X'_1 e} = \frac{1}{k}$.
- (6) E_k and σ_k are the expected value and the standard deviation of return per unit of equity capital, respectively.

The solution of this minimized question, according to Kim and Santomero (1988), obtains the efficient frontier space (E, σ) and portfolio weights X'_1 at each efficient portfolio.¹

2. CAPITAL REGULATION AS THE INSTRUMENT FOR OPERATIONAL RISK CONTROL

Kim and Santomero (1988) believed the global efficient frontier G_0G_2 in Fig. 1 indicates the enveloped curve of efficient frontier under various capital ratios k. The portfolio moving up along the global efficient frontier G_0G_2 represents the smaller of capital ratio k, i.e. the larger of risk and expected return. In the absence of capital regulation, the area under G_0G_2 will be the feasible solutions of the bank. With capital regulation k existing, the feasible solutions for banks will be reduced to the area below R_0R_2 . The existence of capital regulation excludes part of feasible solutions with higher risk and expected return. In order to avoid the bankruptcy probability in excess of a certain given probability α , regulators restrict banks to choose portfolio opportunity at the left of insolvency control line L_R which intersects R_0R_2 at G_1 , so when the regulator sets the ceiling of expected return of banks' equity E_R , it will make

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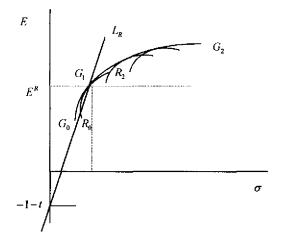


Fig. 1. The global efficient frontier G_0G_2 indicates the enveloped curve of efficient frontier under various capital ratios k. The portfolio moving up along the global efficient frontier G_0G_2 represents the smaller of capital ratio k, i.e. the larger of risk and expected return.

the banks giving up the points under R_0R_2 with rate of equity return higher than E_R , so as to reach the regulation target goal of insolvency probability control. It is then converted to the risk weight of each asset from the ceiling expected return of banks' equity E_R .

In case of bank failure, economic disturbance will cause huge social cost. Assume there is a tolerant cost t, so banks can be allowed to be continued operation even if its capital is exhausted. Assuming government requires a bank to declare insolvency when its capital and tolerant cost is exhausted, i.e. $(E \le -1 - t)$, the setting of bankruptcy probability α by regulator will affect the choice of banks in portfolio.

$$prob(E \le -1 - t) \le a \tag{2}$$

$$E \ge -1 - t - \Phi(\alpha)\sigma \tag{3}$$

 L_R in Fig. 1 indicates the regulation of government on the bankruptcy probability of the bank, and the slope of L_R is, $-\Phi(\alpha)$, $\Phi(\cdot)$ is the inverse of the cumulative standard normal distribution. Left of L_R meets the control level of risks and expected returns-the larger α , the larger its slope, which indicates that the regulation is stricter.

If the regulator converts the given level of bankruptcy probability to the control of risks and expected returns, it will be helpful in proposing effective regulatory policy. In Fig. 1, L_R intersects G_0G_2 at G_1 . Assuming expected return of G_1 portfolio is E^R , the regulator may take E^R as the ceiling of banks' expected return. Then, the banks' behaviors in pursuing risks will be under control.

$$E^{R} \ge E = \left(1 - \sum_{i=1}^{n} x_{i}\right) u_{0} + \left(\sum_{i=1}^{n} x_{i}\right) u_{i}$$
(4)

Assuming $a = [a_i]$ is an $n \times 1$ vector of asset risk weights set by the regulator and, i = 1, 2, ..., n, then for including one unit asset *i*, the bank has to hold at least a_i units of equity capital, i.e. $\underline{a'}\underline{x}_1 \leq 1$, and can hold maximum $(1 - a_i)$ unit of deposit. Since E^R is the ceiling of banks' expected return, the optimal risk weight a_i^k can be obtained by:

$$u_i \le (1 - a_i)u_0 + a_i E^k$$
 (5)

$$a_{i}^{k} \ge \frac{u_{i} - u_{0}}{E^{k} - u_{0}} \quad \text{if } u_{i} - u_{0} > 0$$

$$a_{i}^{k} = 0 \qquad \text{if } u_{i} - u_{0} \le 0$$
(6)

We know from Eq. (5) if $u_i > u_j > u_0$ then $a_i^k > a_j^k$, i.e. an asset with higher expected return has a larger risk weight.

From (6): $\frac{\partial a_i^k}{\partial a} = \frac{\partial a_i^k}{\partial E^R} \cdot \frac{\partial E^R}{\partial a} < 0$ can be found that, when a regulator desires to induce the bank runted probability of bank a, rick weights

tighten regulation and reduce the bankruptcy probability of bank a, risk weights must be raised.

Also, $\frac{\partial a_i^k}{\partial t} = \frac{\partial a_i^k}{\partial E^R} \cdot \frac{\partial E^R}{\partial t} < 0$ can be observed. If the government believes bank

failure will cause huge social cost so to increase tolerant cost, risk weights must be decreased. It means postponing risky banks to be insolvent must pay the expense of raising risk weights.

By that, Kim and Santomero (1988) concluded the following important results: (1) fixed capital regulation has no effect; (2) the theoretical risk weight in risk-based capital regulation is derived; (3) the size of risk weight will affect the asset combination of banks and change the decision of portfolio.

Both of capital regulation and deposit insurance will increase the banks' capital cost and control risk. Kim and Santomero (1988) propose only riskbased capital regulation to lessen the moral hazard under uniform capital regulation but not consider about the relationship between deposit insurance and capital regulation. If we can coordinate both of the policies together, we can complete not only the goal of preventing fraud but also decreasing the resource wasted. In the following of this paper, firstly we base on Kim and Santomero (1988) model and take deposit insurance system into consideration. Then we coordinate both of the policies together and discuss whether the decision of portfolio would be changed and how would the determination of risk weights be affected.

3. DEPOSIT INSURANCE AS AN INDEPENDENT INSTRUMENT FOR BANKRUPT RISK REGULATION

Regulator sets up a deposit insurance system to be participated by banks. Deposit insurance system will protect depositors from a certain amount of loss if banks fail to prevent impact from social financial instability. Since 1990, the global financial crisis appears unexpectedly, depositors fear bank failure more seriously. Banks participating in the deposit insurance system have more attraction for depositors because of safety, so banks will join as members of deposit insurance system voluntarily and take insurance premium as one of necessary costs. As to deposit insurance rate, it depends on the target of regulation. Assuming regulators set E^R as the ceiling rate, fund cost will be increased, the efficient frontier will move down. Whether insurance rate is fixed or variable will influence the effect achieved by the efficient frontier movement. This section discusses how the two different kinds of deposit insurance premium rates, fixed rate and variable rate, influence the bank's portfolio.

3.1. Effect of Fixed-Rate Deposit Insurance on Bank's Portfolio

Assume that a bank participates in deposit insurance, paying premium per unit of deposit at fixed rate d. The funding cost would increase to $u_d = (u_0 + d)$, and deposit insurance institutes assume the insolvency risk completely. Imposing the new funding costs into Eq. (1), a new efficient frontier (E_d, σ_d^2) will be obtained. The expected return of equity is affected by the fixed rate of deposit insurance as follows:

$$\frac{\partial E_d}{\partial d} = -\left(\frac{1/k}{k} - 1\right) < 0 \tag{7}$$

In Fig. 2, if a specific capital ratio k is maintained, the bank participating a fixed rate d deposit insurance system will shift the efficient frontier from R_0R_2 down to D_0D_2 . The higher the fixed rate is, the lower efficient frontier becomes. When k becomes smaller, the distance between D_0D_2 and R_0R_2 will be wider, indicating that in the case of charging with the same deposit insurance premium

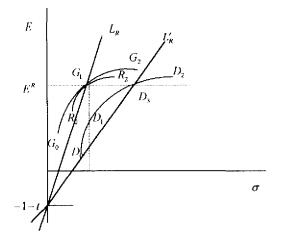


Fig. 2. If a specific capital ratio k is maintained, participating a fixed deposit oinsurance rate d will shift the efficient frontier from R_0R_2 down to D_0D_2 , the higher the fixed rate is, the lower efficient frontier is.

rate, a bank with lower capital will have lower equity return because of higher liability.

The new efficient frontier in Fig. 2 will shift down to D_0D_2 . It means that after participating in the insurance deposit system, the optimal portfolio of a bank will move down from G_1 to D_1 . It indicates that when deposit insurance institutes take the risk of bankruptcy, the bank will have lower equity return and higher risk of bankruptcy, because of higher funding cost.

If the regulator disregards the downward moving of the efficient frontier of banks and maintains the original asset risk weight, then E^* will intersect with efficient frontier D_0D_2 at D_3 , and the risk of banks' operation and bankruptcy will both increase. Therefore, after banks participating in deposit insurance system, if the regulator maintains the original risk-based capital regulating policies, it will drive banks to pursue high-risk operations, and move the insolvency control line L_R rightward to L'_R , meaning the deterioration of bankruptcy probability. This is the moral hazard resulting from banks participating in fixed deposit insurance rate system.

3.2. Effect of Variable-Rate Deposit Insurance on Bank's Portfolio

In fact, both deposit insurance and risk-based capital regulation increase the funding cost of banks at the time of increasing operational risk, so to block

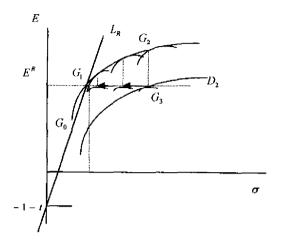


Fig. 3. Banks decrease portfolio of area $G_2G_1G_3$, and efficient frontier shift from $G_0G_1G_2$ to $G_0G_1G_3$ because of variable rate. If banks act as risk averters, they will abandon the right of G_1 on G_1G_3 , and choose G_1 as the optimal solution.

banks' motivation to increase risks. Hence, there is a certain substituting effect between the two instruments. It can be found from Fig. 3 that if the regulator sets insolvency control line at L_R , when a bank does not participate in deposit insurance system, it may choose G_1 portfolio on the efficient frontier $R_0G_1R_2$ with the capital ratio k^R .

On the right-hand side of G_1 on G_1G_2 , each specific k can be found a corresponding deposit insurance premium rate $d_v (\sigma_d, k, E^R)$ satisfying $E_d = E^R$ and $\frac{\partial d_v}{\partial \sigma_d} > 0$. Banks' equity return will decrease to E^R , so as to match target level of capital regulation.²

$$\frac{\partial d}{\partial E^{R}} = -\frac{k}{1-k} < 0, \frac{\partial d_{v}}{\partial k} < 0, \frac{\partial d_{v}}{\partial \sigma_{d}} > 0$$
(8)

It can be concluded that in the absence of capital regulation, taxing higher premium rate on banks with higher operational risk, i.e. adjusting the deposit insurance rates corresponding to the operating risk, would achieve the effect of risk-based capital regulation. As proposed by Sharpe (1978) and Ronn and Verma (1989), given the duality between the insurance premium and the capital ratio, the variable-rate deposit insurance system can be a substitute reciprocally for the risk-based capital regulation system.

4. COMBINING CAPITAL REGULATION AND DEPOSIT INSURANCE AS INSTRUMENTS FOR OVERALL RISK REGULATION

Many countries adopt lower premium rates to attract banks to participate in deposit insurance system. Buser, Chen and Kane (1981) took it as a subsidy from regulator to the banks, and risk-based capital regulation would correct the effect of unfair pricing of the former. This section will discuss the coordination between the two pricing policies, allowing the risk-based capital regulation to control bankruptcy probability ex ante, and the deposit insurance system will protect the interests of the depositors in ante. This will not only achieve the goal of regulator, but also would not result in waste of resources.

4.1. Combining Fixed-Rate Deposit Insurance and Risk-Based Capital Regulation

When fixed insurance rate adopted, assuming $E_d > E^R$, it means the regulator attracts banks to participate in deposit insurance system with a lower premium rate. The risk-based capital regulation should take the existence of deposit insurance into consideration to identify a reasonable capital ratio and the risk weight, so as to achieve objective of bank regulation.

Fixed-rate deposit insurance policy will drive efficient frontier downward. The lower the capital ratio is, the lower the efficient frontier becomes. In Fig. 4, as we move up along the global frontier, the underlying portfolio corresponds to the lower capital ratio, hence, the larger distance between the efficient frontiers before and after participating in deposit insurance system. However, for each fixed rate d, there is always a corresponding capital ratio k, matching the equity return after banks' participation in the insurance system to the expectation of regulator, i.e. $E_d = E^R$.

If the regulator charges only fixed insurance premium rate d_3 on point G_3 in Fig. 4, G_3 will shift down to R_3 and achieve the ceiling rate of capital return E^{κ} set by the regulator, with the corresponding capital ratio k_3 . On the global efficient frontier G_0G_2 , points on the right-hand side of G_3 , in addition to fixed premium rate d_3 , the regulated capital ratio k_3 is needed also to achieve the regulating target goal. Points on the left-hand side of G_3 represent lower-operational-risk portfolio and the regulator imposes no restriction. Under deposit insurance system, they play as the roles of subsidizing high-risk banks and the equity returns will be lower than E^{κ} .

Hence, fixed deposit insurance rate system will cause the bank to have higher funding cost. It eliminates only part of high-risk portfolio, but can still lighten

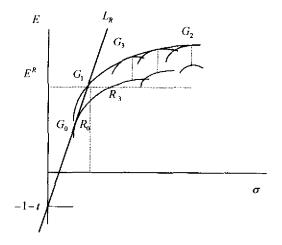


Fig. 4. Point G_3 represents only fixed insurance premium rate d_3 is charged, portfolio will be shifted down to R_3 achieve the ceiling rate of capital return E^R as set by the regular, at the same time the corresponding capital ratio is k_3 . On the global efficient frontier G_0G_2 , points at the right of G_3 , in addition to levying fixed premium rate d_3 , the

regulated capital ratio k_3 is needed also to achieve the regulating target goal.

the responsibility of capital regulation. So the capital ratio will decrease to the level of portfolio G_3 and the optimal portfolio will move right from G_1 to R_3 . At this time, the slope of banks' bankruptcy control line will become flatter, i.e. higher bankruptcy probability, because fixed-rate system makes the equity returns of portfolios on G_1G_3 lower than E^R . The risk-based capital regulation will not be a limit to it, so the feasible solutions of portfolios will increase with the higher risk $R_0G_1R_3$ area, which leads to riskier optimal solution.

The higher the fixed deposit insurance rate is, the larger the distance between G_1 and R_3 , and the area of $R_0G_1R_3$ becomes larger. The funding cost of bank increases as the result that fixed deposit insurance rate must make it up with higher return on higher risk. Further, since the premium rate is not increasing with the hike of risk, it gives incentives to a bank to increase risky assets. Since risk-based capital regulation can only counter off part of high-risk portfolio, risk of the optimal solution is boosted up.

4.2. Combining Variable-Rate Deposit Insurance with Risk-Based Capital Regulation

When variable rate is adopted, assuming $E_d > E^R$, it means that the regulator attracts banks to participate in deposit insurance system with lower premium

rate. The risk-based capital regulation should be reconsidered with the existence of deposit insurance in setting reasonable capital ratio and risk weight to reach regulation target goal.

We have discussed in Section 3, each specific k can find a corresponding deposit insurance premium rate $d_v(\sigma_d, k, E^R)$ satisfying $E_d = E^R$. Banks' equity return will be decreased to E^3 , which meets with the capital regulation target goal completely, hence variable-rate deposit insurance can substitute risk-based capital regulation.

If the regulator taxes insufficient variable premium rate from banks, i.e. $\underline{d}(\sigma_d) < d_v, \underline{d}' > 0$, to attract the banks participating in insurance system, then $E > E_d > E^R$, although there is a slight collapse in banks' equity return, but not reaching the E^R regulating level. As shown in Fig. 5, regulator charges variable-rate premium at the right of G_1 on the curve G_1G_2 . On the same global efficient frontier, the larger σ is, the higher the deposit insurance rate $\underline{d'}|_{\sigma_d}$ is taxed, i.e. the flatter efficient frontier. When $d = d_v$, the efficient frontier will become a horizontal line and the regulation target goal will be reached even though capital regulation not existing or required. The efficient frontier between G_1G_2 and G_1D_3 will still need the assistance of capital regulation to reach regulation target goal, as the efficient frontiers are all started from G_1 , hence as long as the

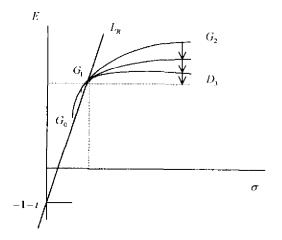


Fig. 5. Regulator charges variable-rate premium at the right of G_1 on the curve G_1G_2 . On the same global efficient frontier, the larger σ is, the hight the deposit insurance rate $\underline{d'}|_{\sigma_d}$ is taxed, i.e. the flatter efficient frontier. When $\underline{d} = d_1$, the efficient frontier will become a horizontal line and the regulation target goal will be reached even though capital regulation not existing.

variable-rate premium is charged insufficiently, original regulatory instrument, capital ratio, must be injected to reach the regulating target goal.

4.3. Modification of Risk Weights

If the regulator, after banks participating in the deposit insurance system, still hope to maintain the original equity return ceiling, E^{R} , he needs to reconsider new asset risk weights.

In Fig. 2, L'_R intersects with D_0D_2 at D_3 , the regulator must set new asset risk weight based on new funding cost. From Eq. (6), the optimal risk weight α^*_i can be obtained.

$$a_i^* \ge \frac{u_i - u_d}{E^R - u_d} \quad \text{if } u_i - u_d > 0 \tag{9}$$
$$a_i^* = 0 \qquad \qquad \text{if } u_i - u_d \le 0$$

If $a_i^k > 1$, then $a_i^* > a_i^k$. If $a_i^k < 1$, then $a_i^* < a_i^k$. It means after the banks participating in deposit insurance system, regulator must raise risk weights of risky assets and reduce risk weights of low-risk assets to maintain the upper bound of original equity return, E^k . Under the circumstances that the funding cost of banks increases, and the risk of bankruptcy is protected by insurance institutes in meeting claims, the incentive to high-risk assets will increase. Hence, through modification of risk weight and increasing banks' costs in taking risk, the target goal of regulating upper bound of equity return, E^R , can be achieved.

The level of risk weights needs to be adjusted because of higher operational risks. Whether it will be different between prior and after bank participating in the insurance system, depends if the insurance premium rate is variable or not.

$$\frac{\partial a_{i}^{*}}{\partial \sigma_{d}} = \frac{\partial a_{i}^{*}}{\partial E^{R}} \cdot \frac{\partial E^{R}}{\partial \sigma_{d}} + \frac{\partial a_{i}^{*}}{\partial d_{v}} \cdot \frac{\partial d_{v}}{\partial \sigma_{d}}$$
(10)

As shown in Eq. (10), if the rate is fixed, the second term of Eq. (10) would be zero. There would not be any difference between prior and after participating in deposit insurance system with the level of risk weights needing to be raised because of higher operational risk. Banks increase funding cost and operational risk but not necessarily increase additional risk weight. And, since there is the insurance institute to shelter a bank from the bankruptcy risk, it certainly creates motive in holding high-risk assets. Therefore, the results of this section,

that combining risk-based capital regulation and fixed-rate insurance system can reach the original target goal of regulators, are the same as argument in Section 4.1.

If the rate is variable, and the banks are holding high-risk assets, i.e. $a_i^k > 1$, the second item of Eq. (10) would be larger than 0. Thus, banks have to pay extra prices for higher operational risk, increasing risk weight and insurance premium rate, because the increase of variable rate can offset the increase of equity return. Therefore, combining risk-based capital regulation and variable-rate insurance system will reach the original target goal of regulators, same as the result in Section 4.2.

4.4. Summary

Capital regulation and deposit insurance are both the instruments for regulator used to control the operational risks and insolvent probability of a bank. Examining the effects and reciprocal influences of the policies would enhance the effectiveness of decision and reduce wasting of resources. The main results in this section are as follows:

- (1) By raising capital ratio in holding risky asset and excluding high-risk portfolio, risk-based capital regulation will attain the regulatory target goal of controlling the bankruptcy probability.
- (2) If risk-based capital regulation is replaced with fixed deposit insurance premium, the banks would pursue high operational risk. It would cause moral hazard, if the original regulation target goal were to be achieved in this manner.
- (3) In the absence of capital regulation, imposing high insurance premium on the banks with high operational risk, i.e. pricing insurance premium based on operational risks would obtain the same effect as risk-based capital regulation. So the variable-rate deposit insurance system can be a substitute for risk-based capital regulation.

If the regulator decides to charge lower insurance premium to attract banks to participate in the deposit insurance system, the subsidy on the banks under deposit insurance system will be corrected by risk-based capital control. The coordinate situations are as follows:

(1) The increasing capital cost resulting from fixed insurance premium has to be compensated with higher return on higher risk. However, the insurance premium rate does not increase with high risk, which strengthens the incentive to increase the risk-taking of banks. Risk-based capital regulation can only eliminate part of the high-risk portfolio, and the risk of optimal solution would still be increased. (2) If the variable-rate insurance premium is insufficient, it may cope with the original capital ratio regulation, but the risk weight must be adjusted as raising the risk weights of high-risk assets. Since the raised weight can offset completely the increased equity return resulting from insufficient premium, the banks will not have the incentive to increase the risk taking and the original regulatory target goal will be reached. Conversely, if the regulator fails to adjust the risk weight before adopting the insufficient variable-rate insurance premium system, the increased equity return due to insufficient premium would not be offset. So the tradeoff between bank's return and risk can be improved, which would increase the incentive for banks to take risks.

5. EMPIRICAL ANALYSIS OF OPERATIONAL AND BANKRUPT RISK MANAGEMENT

5.1. Formation of Hypothesis

According to the evolution of capital regulation and deposit insurance in Taiwan during the period from 1985 to 2000 (see Table 1), we classify the period into three stages. (1) The insurance rate had experienced three times reduction during the period of fixed rate system. (2) Risk-based capital

Times	Regulatory changes and interactions	Interactions	Possible effects on bank return and risk
1982.09	Implement fixed-rate deposit insurance system, rate set at 0.05%	Increase bank capital cost	Lower risk to return ratio
1987.01	Lower insurance rate to 0.04%	Decrease bank capital cost	Higher risk to return ratio
1988.01	Lower insurance rate to 0.015%	Decrease bank capital cost	Higher risk to return ratio
1993.01	Implement RBCR	Eliminate high risk portfolio	Higher risk to return ratio
2000.01	Adopt variable deposit insurance rate	Eliminate high risk portfolio	Higher risk to return ratio

Table 1. Recent Regulatory Changes and Interactions in Taiwan.

regulation policy is executed overall in January 1993. (3) Variable rate deposit insurance rate was introduced in January 2000.

Based on the results of Section 4.4, the implementation of new regulation may lead to changes of bank portfolio decision. (1) Moral hazard may exist under fixed-rate deposit insurance system, so the higher the insurance rate is, the lower the banks' return to risk ratio becomes. (2) Implementation of riskbased capital regulation can achieve the goal of reducing bankruptcy, so the banks' return to risk ratio can be higher. (3) If the regulator did not raise the risk weights but charged insufficient variable-rate premium rate, the increased equity return due to insufficient premium would not be offset. So the tradeoff between bank's return and risk can be improved. From the proceeding discussion, this paper forms the following hypotheses to test whether the tradeoff between banks' return and risk can be changed or argued before and after adopting new regulation.

- (1) Under the fixed-rate deposit insurance system, the higher the rate is, the lower the bank's return to risk ratio is.
- (2) After risk-based capital regulation is imposed the bank's return to risk ratio can be improved.
- (3) If the regulator fails to raise the risk weights when adopting insufficiently variable-rate insurance premium system, the banks' return to risk ratio will be higher, and banks will have more incentives to take risk.

5.2. Data and Samples

The deposit insurance system was implemented in September 1985 in Taiwan, with the fixed premium rate set at 0.05%. At that time, the regulator had many restrictions on the establishment and operation of banks, and the banks operated rather conservatively and hardly went bankrupt, so not many banks participated in the deposit insurance. Latterly, in order to attract participants, the regulator adopted low rate strategy, and lowered the insurance premium rate to 0.04% and 0.015% in January 1987 and January 1988 respectively.

Following the efforts of Taiwan joining WTO, the paces of financial liberation and internationalization have been accelerating, and the regulator eased the restriction on establishment and operations of banks. The competitions between banks are getting more intense, so the operational strategies become more opened than ever, and the risks faced by banks are much higher. Risk-based capital regulation policy was executed completely in January 1993, requiring the capital level set in accordance with the risk structure of banks' assets. We can see asset items and corresponding risk weights regulated in Taiwan from Table 2. The constituents of capital including three categories

regulated in Taiwan shown in Table 3. These rules are based mainly from Amendment to the Capital Accord to Incorporate Market Risks proposed in 1996 by Basle Committee on Banking Supervision. The amount of risky assets is determined by asset balances multiplying corresponding risk weights, bank capital must exceed 8% of risky assets, even variable deposit insurance rate was implemented in January 2000 to control the banks' risks through price mechanism and maintain the financial stability.

This article examines the effects of bank capital regulation and deposit insurance policies on the banks' return to risk ratios during the period from 1985 to 2000. The tradeoff may be regarded as one kind of risk premium. Ronn and Verma (1989) and Mei-Ying Liu (1994) took market value of equity as the

Risk weight %	Asset item	
0	Cash, claims on central government and central banks denominated in national currency and funded in that currency, Claims on OECD central government and central banks, other claims on OECD central governments and central banks, Claims collateralised by cash of OECD central- government securities or guaranteed by OECD central governments	
10	Claims on domestic public-sector entities, excluding central government, and loans guaranteed by such entities	
20	Claims on multilateral development banks and claims guaranteed by, or collateralised by securities issued by such banks, Claims on banks incorporated in the OECD and loans guaranteed by OECD incorporated banks, Claims on banks incorporated in countries outside the OECD with a residual maturity of up to one year and loans with a residual maturity of up to one year guaranteed by banks incorporated in countries outside the OECD, Claims on non-domestic OECD public-sector entities, excluding central government, and loans guaranteed by such entities, Negotiation of export draft, Inward remittance, Claims guaranteed by government- authorized credit institutes,	
50	Loans fully secured by mortgage on residential property that is or will be occupied by the borrower or that is rented	
100	Claims on banks incorporated outside the OECD with a residual maturity of over one year, Claims on central governments outside the OECD (unless denominated in national currency – and funded in that currency –see above), all other assets	

Table 2. Risk Weights and Asset Items.

Source: Ministry of Finance, Taiwan.

Eligible capital	Capital items	
Tier 1 capital	Common stock, non-cumulative preference shares, subscription capital, share premiums, retained profit, general reserves, legal reserves, and minority interests in the equity of subsidiaries less than wholly owned, excluding revaluation reserves and cumulative preference shares	
Tier 2 capital	Cumulative preference shares, revaluation reserve of fixed asset, a 55% discount on hidden values of revaluation reserves in long-term holdings of equity securities, convertible bond, general loan-loss reserve, allowance for bad debt, and long-term subordinated debt	
Tier 3 capital	Short-term subordinated debt	

Table 3. Classifications of capital.

Source: Ministry of Finance, Taiwan.

basis of computing risk and equity return. This paper will take expected stock return rate and variance of stock return rate as instrument variables of the banks' equity returns and risks respectively. It takes the publicly held banks as samples from 1985 to 1999 with full records of participating in deposit insurance system. It includes Changhua Commercial Bank, Huanan Commercial Bank, First Commercial Bank, China Trust Commercial Bank, Hsinchu Business Bank, Taipei Business Bank, Tainan Business Bank, Kaohsiung Business Bank, Taitung Business Bank and Taichung Business Banks, totaling ten in number. The stock return rate data were taken from the Economic Statistical Database of AREMOS\UNIX of Ministry of Education, Taiwan. During the period of adopting fixed-rate system, the Deposit Insurance Company had reduced the rate for 3 times. In January 1993, the risk-based capital regulation was imposed, and variable-rate deposit insurance rate was introduced in January 2000. This paper takes the monthly stock return rate of the 10 banks before and after the events to calculate the expected value and variance, then test the recently rederegulatory impact of bank capital and deposit insurance on risk attitude change of banks in Taiwan.

5.3. Empirical Model

This paper uses the general linear regression of the least square model to test the following hypotheses.

Hypothesis I: Under fixed-rate deposit insurance system, the higher the insurance premium rate is, the lower the bank's return to risk ratio is.

With a dummy variable to represent before and after reduction of rates, the regression equation is designed as follows:

$$r_i = \alpha_0 + \alpha_1 \sigma_i^2 + \alpha_2 I + \alpha_3 (\sigma_i^2 \cdot I) + \alpha_4 (\sigma_i^2 \cdot T) + \alpha_5 (I \cdot T) \varepsilon_i$$
(11)

where,

- r_i is the expected stock return rate of the *i*th bank.
- σ_i^2 is the variance of stock return rate of the *i*th bank.
- I is the dummy variable = 0 representing after the reduction of rate.

= 1 representing before the reduction of rate.

- T is the dummy variable = 1, 2, 3 representing 1985, 1987 and 1988 respectively.
- ε_i is the residual error.

If α_3 is less than 0 significantly, we cannot reject Hypothesis I. After the reduction of insurance premium rate, there is an improvement in the tradeoff between banks' return and risk.

Hypothesis II: After implementation of risk-based capital regulation, the banks' return to risk ratio can be higher.

The dummy variable represents before and after the implementation of riskbased capital regulation, the regression equation can be as follows:

$$r_i = \beta_0 + \beta_1 \sigma_i^2 + \beta_2 C + \beta_3 (\sigma_i^2 \cdot C) + \varepsilon_i$$
(12)

Where, C is the dummy variable = 0 representing after the implementation of risk-based capital regulation.

=1 representing before the implementation

of risk-based capital regulation.

 ε_i is the residual error.

If β_3 is less than 0 significantly, we cannot reject Hypothesis II. Which implies that after the implementation of risk-based capital regulation, the banks' return to risk ratio can be higher.

Hypothesis III: If the regulator did not raise the risk weights but charged insufficient variable-rate premium rate, the banks' return to risk ratio will be higher, giving more incentives for bank to take risks.

With the dummy variable representing before and after adopting variable-rate insurance premium, the regression equation may be further set up as follows:

$$r_i = \gamma_0 + \gamma_1 \sigma_i^2 + \gamma_2 V + \gamma_3 (\sigma_i^2 \cdot V) + \varepsilon_i$$
(13)

Where V is the dummy variable = 0 representing after the implementation of variable-rate premium system.

=1 meaning before the implementation of

variable-rate premium system.

 ε_i is the residual error.

If γ_3 is less than 0 significantly, then we cannot reject Hypothesis III, after implementation of variable premium rate system, the banks' return to risk ratio will be higher and inspire banks to take risk.

5.4. Empirical Results

The insurance deposit system in Taiwan had experienced three times reduction during the period of fixed rate system. We take six months period before and after the event, the reduction of premium rate, to test whether the tradeoff between return and risk of 10 banks has changed. Firstly, we test the coincidence of regression lines. If we reject the hypothesis, all of α_2 , α_3 , α_4 , α_5 equal to 0, it represents the regression lines do not coincide, i.e. the intercepts or slopes of regression lines before and after the event are not equal. It means dummy variables have explanatory ability. Secondly, we test the parallel of regression lines. If we reject the hypothesis, both of equal to 0, it represents the slope of regression lines are not equal, i.e. the regression lines do not parallel. It means there is interaction between dummy variables and independent variables. F-values are 38.2944** and 4.3360* respectively. It indicates both of rate-reduction and period factors may change the tradeoff between return and risk significantly. In order to prohibit heteroscedasticity, following constant variances assumption in regression model, this paper adopts weighted least squared model, taking the square of stock return rate variance σ_i^2 as the weight, and induced the parameter estimates of regression coefficient after transformation and *t*-value, with results as follows:

$$r_{i} = 0.6319 + 0.0244\sigma_{i}^{2} = 4.7469I - 0.0540(\sigma_{i}^{2} \cdot I) + 0.0071(\sigma_{i}^{2} \cdot T) + 3.7794(I \cdot T) + \varepsilon_{i}$$

$$(4.328^{**}) (0.852) (-7.945^{**})(-2.896^{**}) \quad (0.668) \quad (5.969^{**}) \quad (14)$$

$$F = 46.740^{**} \qquad R^{2} = 0.8123$$

We can see from Eq. (14), α_2 is significantly negative, it indicates after the reduction of fixed rate, the regression line moved up, with less insurance expenditure, and increase the return to the bank significantly. And, α_3 is significantly negative, it means after the reduction of premium rate, there is improvement in the tradeoff between banks' returns and risks, hence

Hypothesis I can not be rejected. However, $\alpha_4 > 0$ is not significant, we still can see that between 1985 and 1988, Taiwan was in the stage of high economic growth, the tradeoff between bank's returns and risks is gradually improved. Besides α_5 is significantly positive, it means that there is interaction between time factor and premium rate factor. The later the period, economic growth in Taiwan is higher, the descending premium expenditure is more insignificant to the banks' return.

Next, it follows that with the six-month period before and after the riskbased capital regulation for the examination of 10 banks prior and after the event, the changes of tradeoff between return and risk. The coincidence and parallel of regression equation is examined firstly. The values of F are 26.2458** and 10.6593** respectively, indicating that the risk-based capital regulation may have changed significantly the tradeoff between the banks' return and risk. The parameter estimate and t-value of regression coefficient are as follows:

$$r_{i} = -0.0239 + 0.0262\sigma_{i}^{2} - 0.2621C - 0.0437(\sigma_{i}^{2} \cdot C) + \varepsilon_{i}$$

$$(-0.077) (5.005^{**}) \quad (-0.609) (-0.0039)$$

$$F = 67.697^{**} \qquad R^{2} = 0.9104^{**}$$
(15)

It can be found from Eq. (15), β_1 is significantly positive, so there is a positive relationship between risk and return. We also find β_3 is significantly negative. It represents after the implementation of risk-based capital regulation, the banks' return to risk, so Hypothesis II cannot be rejected.

Finally, we take six months period before and after the event, implementation of variable premium rate system, to test whether the tradeoff between return and risk of 10 banks has changed. The coincidence and parallel of regression line is tested firstly. *F*-values are 9.5836** and 0.6180 respectively. It indicates after the implementation of variable-rate premium system, the banks' return level may have been changed significantly. The parameter estimate of regression coefficient and t-value are as follows:

$$r_{i} = -1.7761 + 0.1727\sigma_{i}^{2} - 0.204V - 0.1108(\sigma_{i}^{2} \cdot V) + \varepsilon_{i}$$

$$(-2.028^{*}) (1.527) (-0.106) (-0.444)$$

$$F = 2.83 \qquad R^{2} = 0.3466 \qquad (16)$$

From Eq. (16), we can see that the null hypothesis $\gamma_3 = 0$ is true. If we abandon interaction term ($\sigma_i^2 \cdot V$), retest the coincidence of regression lines, *F*-value is 6.8539*. It represents after implementation of variable rate, the return level of bank may have been changed significantly. The parameter estimate of regression coefficient and t-value are as follows:

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$$r_i \approx -1.61 + 0.15\sigma_i^2 - 1.037V + \varepsilon_i$$

(-2.08*) (2.618*) (-1.524)
$$F = 4.351^* \qquad R^2 = 0.3386 \qquad (17)$$

From Eq. (17), we can see that the null hypothesis $\gamma_2 = 0$ is not rejected, it represents after implementation of variable rate, the level of banks' return has been changed insignificantly.

6. CONCLUSIONS AND MANAGEMENT IMPLICATION

This paper based on the mean variance model on bank portfolio decision by Kim and Santomero (1988), modifies and proposes a model discussing the coordination of capital regulation and deposit insurance to reach the target goal of regulator. The major theoretical results are as follows:

- (1) Risk-based capital regulation with the increased capital ratio for holding risky assets can exclude high-risk portfolios. It has achieved the regulation target goal of controlling the insolvency probability of banks in Taiwan during the period of 1985 to 2000.
- (2) When fixed-rate deposit insurance system is applied in place of risk-based capital regulation, the banks pursue high-risk operation to attain the original regulation target goal and cause moral hazard during this period.
- (3) Adopting variable-rate insurance premium makes banks have the same effect as risk-based capital regulation. So, variable-rate deposit insurance system can substitute for risk-based capital regulation during this period.

If the regulator takes a lower insurance premium rate to attract banks to participate in the deposit insurance system, the subsidy on the banks under deposit insurance system might be corrected by risk-based capital control. The coordinate situations are as follows:

- (1) The increasing capital cost resulted from fixed insurance premium has to be compensated with higher return on higher risk. However, the insurance premium rate does not increase with high risk, which strengthens the incentive to increase the risk taking of banks. Risk-based capital regulation can only eliminate part of the high-risk portfolio, and the risk of optimal solution would still be increased.
- (2) If the variable insurance premium is insufficient, the risk weights must be raised to achieve the original regulation effect. Conversely, if the regulator fails to adjust the risk weight before adopting the insufficient variable-rate insurance premium system, so the tradeoff between bank's returns and

risks can not be improved, which would give the incentive for banks to take risks. On the other hand, if the regulator failed raising risk weight, but implemented insufficiently variable insurance premium rate system, it would improve the tradeoff between return and risk if banks can raise the incentive for banks to increase their risk taking.

This article examines the effects of bank capital regulation and deposit insurance policies on the banks' return to risk ratios during the period between 1985 and 2000. It took the publicly-held banks during the period between 1985 and 1999 with full records of participating in deposit insurance system as samples. In Hypothesis I, we test whether higher fixed insurance premium will worsen the tradeoff between banks' returns and risks. In Hypothesis II and III, we test separately whether implementing risk-based capital regulation or variable insurance premium system will improve the tradeoff between banks' returns and risks. Hypothesis I and II are both supported by the empirical results. However Hypothesis III is not supported. It is possible that since most of the data are from governmental banks, they tend to be conservative in adjusting strategies for risk and return, so Hypothesis III is rejected.

To sum up, there is a converse relationship between variable deposit insurance system and risk-based capital regulation, decrease of one rate can substitute for increase of another in theory, each of them has its own advantage to the regulator. Both of them helping each other to achieve the common regulation target goal shall be the most feasible measure. Therefore, considering both of them into pricing strategy simultaneously can reduce redundant taxes on banks and make allocation of resources optimal.

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APPENDIX

According to Kim and Santomero (1988), derivation of the efficient frontier from Eq. (1) result in:

$$E_k = E_{k,m} + \left(\frac{1}{D}\right) [WD(\sigma_k^2 - \sigma_{k,m}^2)]^{\frac{1}{2}}$$

where, $(E_{k,m}, \sigma_{k,m}^2)$ represents the minimum-variance portfolio for a given k.

$$E_{k,m} = \binom{1}{k} \binom{A}{D} + \binom{1}{k} - 1 \binom{1}{D} (DF - AG) - \binom{1}{k} - 1 u_0$$

$$\sigma_{k,m}^2 = \binom{1 - 1}{k}^2 (\sigma_0^2 - H) + \binom{1}{D} \left[\frac{1}{k} - \binom{1}{k} - 1 \right]^2$$

and

$$B = \underline{U}_{1}' \underline{V}_{2}^{-1} \underline{U}_{1} > 0 \quad A = \underline{e}' \underline{V}_{2}^{-1} \underline{U}_{1}$$

$$D = \underline{e}' \underline{V}_{2}^{-1} \underline{e} \quad W = BD - A^{2} > 0$$

$$F = \underline{U}_{1}' \underline{V}_{2}^{-1} \underline{V}_{1} \quad G = \underline{e}' \underline{V}_{2}^{-1} \underline{V}_{1}$$

$$H = \underline{V}' \underline{V}_{2}^{-1} \underline{V}_{1} > 0$$

$$d_{\nu} = \frac{1}{1-k} \left(\frac{A}{D}\right) + \left(\frac{1}{D}\right) (DF - AG) - \frac{k}{1-k} E_{d,m} - u_0$$

$$\frac{\partial d_{v}}{\partial k} = k \left(E_{d,m} - \frac{A}{D} \right) + \frac{A}{D} - \frac{E_{d,m}}{1-k} < 0$$

$$\frac{\partial d_{\nu}}{\partial \sigma_d} = \frac{k}{1-k} \left(\frac{1}{D}\right) (WD)^{1/2} > 0$$